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ON THE COMPLETENESS OF A SYSTEM OF HOMOGENEOUS SOLUTIONS OF THE PLATES THEORY

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The completeness of homogeneous (elementary) solutions in the space of solutions of equations of the elasticity theory with finite energy is proved. The scheme for this proof includes the case of plates inhomogeneous through the thickness and is similar to the scheme given in [1]. One of the general representations of the solution of the elasticity theory equations which are inhomogeneous in the thickness, on whose basis the system of elementary solutions is determined, is presented without proof. This system has been obtained in [2, 3] by another method.

The problem of the completeness of homogeneous solutions was formulated in different aspects [5] by Lur'e [4] in connection with the foundation of one of the versions of the asymptotic method [6, 7]. The connection of this problem with the problem of n-tuple completeness of Keldysh [8] is given and one of the methods to solve it is proposed, which is realized in [9, 10] for the case of the plane and axisymmetric problems.

1. Let $\Omega = S \times [-h, h]$ be the domain occupied by a plate, where $2h$ is the plate thickness, S is its middle surface, ∂S is the boundary of S, $S^{\pm}$ are the plate endfaces corresponding to $x_3 = \pm h$, $\Gamma = \partial S \times [-h, h]$ is the lateral surface. The properties of the plate material are given by the Lamé elastic characteristics $\lambda = \lambda(x_3)$, $\mu = \mu(x_3)$.

The elastic equilibrium of a plate described by the equations

$$\sigma_{ij,j} = 0, \quad \sigma_{ij} = \lambda\delta_{ij}u_{l,l} + \mu(u_{i,j} + u_{j,i}) \tag{1.1}$$
$$\mathbf{u} = \{u_1, u_2, u_3\}, \quad x = \{x_1, x_2\} \in S, \quad x_3 \in [-h, h]$$

is considered.

A system of elementary solutions satisfying the following homogeneous conditions on the plate endfaces is presented in [2, 3]

$$\sigma_{i3}\big|_{S^{\pm}} = 0 \quad (i = 1, 2, 3) \tag{1.2}$$

For the later discussion, it is convenient to determine the homogeneous solutions constructed in [2, 3] by using the general representation of the solution of (1.1) and (1.2), whose form we present without proof.

Namely, every solution of the elasticity theory equations (1.1) which satisfies the boundary conditions (1.2) can be represented as

$$u_i = u_i^{(1)}(\Phi_0, \Phi_1) + u_i^{(2)}(c) + u_i^{(3)}(g) \tag{1.3}$$

$$u_\alpha^{(1)} = \psi_\alpha + \partial_\alpha \Big[e\Phi_1 - \sum_{i=0}^{1} (x_3{}^i \Phi_i - q_i(x_3)\,\Delta\Phi_i) \Big] \tag{1.4}$$

$$u_3{}^{(1)} = \Phi_1 + (k_1 - ek_0)\,\Delta\Phi_1 - (\lambda^* + \mu^{(0)})^{-1}\mu^{(0)} k_0 \Delta\Phi_0$$

$$u_\alpha{}^{(2)} = \partial_\alpha b, \quad u_3{}^{(2)} = -b' - 2q\Delta c', \quad b = pc'' - r\Delta c \tag{1.5}$$

$$\alpha = 1, 2, \quad (\cdot)' = \partial_3(\cdot)$$

$$u_1{}^{(3)} = \partial_2 g, \quad u_2{}^{(3)} = -\partial_1 g, \quad u_3{}^{(3)} = 0 \tag{1.6}$$

Here $\Phi_i = \Phi_i(x)$ are biharmonic functions, ψ_1, ψ_2 are conjugate harmonic functions connected with Φ_0 by the relationship

$$\partial_1\psi_1 = \partial_2\psi_2 = k\Delta\Phi_0$$

$$\mu^{(0)} = \int_{-h}^{h} \mu\, dx_3, \quad \lambda^* = \int_{-h}^{h} \frac{2\lambda\mu}{\lambda + 2\mu}\, dx_3, \quad k = \frac{\lambda^* + 2\mu^{(0)}}{2(\lambda^* + \mu^{(0)})}$$

$$q_0(x_3) = (2k - 1)\, q_2(x_3) - 2k \int_0^{x_3} \mu^{-1}\, dx_3 \int_{x_3}^{h} \mu\, dx_3$$

$$q_1(x_3) = eq_2(x_3) - \int_0^{x_3} k_1\, dx_3 - \int_0^{x_3} \mu^{-1}\, dx_3 \int_{x_3}^{h} x_3 a(x_3)\, dx_3$$

$$q_2(x_3) = \int_0^{x_3} k_1(x_3)\, dx_3 + \int_0^{x_3} \mu^{-1}\, dx_3 \int_{x_3}^{h} a(x_3)\, dx_3$$

$$a(x_3) = \frac{2\mu(\lambda + \mu)}{\lambda + 2\mu}, \quad k_i(x_3) = \int_0^{x_3} x_3{}^i \frac{\lambda}{\lambda + 2\mu}\, dx_3$$

$$e = \frac{a^{(1)}}{a^{(0)}}, \quad a^{(i)} = \int_{-h}^{h} x_3{}^i a(x_3)\, dx_3, \quad \partial_\alpha = \frac{\partial}{\partial x_\alpha}$$

$$\Delta = \partial_1{}^2 + \partial_2{}^2$$

Furthermore, the function $c(x, x_3)$ satisfies the following equation and boundary conditions

$$L(\Delta)\, c(x, x_3) \equiv (\Delta^2 - 2\Delta F + V)\, c(x, x_3) = 0 \tag{1.7}$$

$$F\varphi = \{-p^{-1}[(p\varphi')' - r''\varphi],\ \varphi(\pm h) = 0\}$$

$$V\varphi = \{p^{-1}(p\varphi'')'',\quad \varphi(\pm h) = 0 = \varphi'(\pm h)\}$$

$$p = \frac{\lambda + 2\mu}{4\mu(\lambda + \mu)}, \quad q = \frac{1}{2\mu}, \quad r = \frac{\lambda}{4\mu(\lambda + \mu)}$$

and, finally, we have for the function $g(x, x_3)$

$$L_1(\Delta) g \equiv (\Delta + T) g(x, x_3) = 0 \tag{1.8}$$

$$Tl = \{-\mu^{-1}(\mu l')',\ l'(\pm h) = 0\} \tag{1.9}$$

We consider the functions $c(x, x_3)$ and $g(x, x_3)$ below as functions $c(x)$, $g(x)$ with values in some Hilbert space X, and the operators F, V as unbounded self-adjoint operators in this space.

Following [6, 7], we designate $\mathfrak{u}^{(1)}$, $\mathfrak{u}^{(2)}$, $\mathfrak{u}^{(3)}$, respectively, as the biharmonic, potential, and vortex solutions. Let us emphasize that each satisfies Eqs. (1. 1) and the boundary conditions (1. 2). If λ, $\mu = \text{const}$, then (1. 7) is converted into a biharmonic equation, (1. 8) into a harmonic equation, and (1. 4) into the Lur'e [4] formulas.

2. Let us introduce the concept of elementary solutions for (1. 7) and (1. 8), respectively.

We call a solution of (1. 7) of the form

$$c_k = \sum_{s=0}^{p-1} m_{ks}(x)\varphi_{ks} \tag{2.1}$$

an elementary solution of the first kind. In (2. 1) β_k is the eigenvalue, φ_{k0} is a generalized eigenvector (see [11]) of the operator bundle

$$L(\beta)\varphi \equiv (\beta^2 I - 2\beta F + V)\varphi = 0 \tag{2.2}$$

and φ_{ks} are associated vectors determined from the equations

$$L(\beta_k)\varphi_{ks} + \frac{\partial L}{\partial\beta_k}\varphi_{ks-1} + \frac{1}{2}\frac{\partial^2 L}{\partial\beta_k^2}\varphi_{ks-2} = 0$$

The functions $m_{ks}(x)$ satisfy the equations

$$(\Delta - \beta_k) m_{k0} = 0$$

$$(\Delta - \beta_k) m_{ks} = m_{ks-1} \quad (s = 1, \ldots, p-1)$$

We shall omit the second subscript in the case where there are no associated vectors.

We note that the spectral problem (2. 2) has been studied in detail in [1], where in particular, the double completeness of the system of eigen- and associated vectors is established by using the results in [11].

We now define an elementary potential solution as a vector function of the form

$$u_k^{(2)} = u^{(2)}(c_k)$$

We call every solution of Eq. (1. 8) of the form

$$g_t = n_t(x)\, l_t, \quad (\Delta - \gamma_t)\, n_t(x) = 0, \quad T l_t = \gamma_t l_t$$

an elementary solution of the second kind, where γ_t is an eigenvalue, l_t is an eigenvector of the operator T defined by (1. 9). Evidently T is a positive operator, and therefore all $\gamma_t \geqslant 0$. From the general theory of self-adjoint operators [12], there follows that the system of eigenvectors $\{l_t\}$ forms an orthonormalized basis of the space X_μ, i.e.

$$(l_t, l_s)_{X_\mu} = \int_{-h}^{h} \mu l_t l_s\, dx_3 = \delta_{ts}$$

We define the elementary vortex solution as a vector-function of the form

$$u_t^{(3)} = u^{(3)}(g_t) \tag{2.3}$$

It can be seen by direct substitution that $\gamma_0 = 0$ is an eigenvalue of the operator T. Its corresponding elementary vortex solution is a particular case of the biharmonic solution. Hence, we understand the vortex solution below to be a set of elementary solutions of the form (2. 3) which correspond to points of the spectrum $\gamma_t > 0$ $(t = 1, 2, \ldots)$.

3. The following functional spaces will be used below:

1. X is the space of functions, square integrable in the segment $x_3 \in [-h, h]$ and with weight $p(x_3)$;

2. X^α is the scale of the Hilbert spaces [13] which is obtained by closure of the intersection of the domains of definition $D(V^n)$ of the operators V^n $(n = 1, 2, \ldots)$ in the metric

$$\|\varphi\|^2_{X^\alpha} = \|V^\alpha \varphi\|^2_X$$

3. $L_2(\partial S, X^\alpha)$ is the space of functions with values in X^α and the norm

$$\|f\|^2_{0,\alpha} = \int_{\partial S} \|f\|^2_{X^\alpha} ds$$

4. $W^{(\beta)}(\partial S, X)$ is the space of functions with values in X whose generalized derivatives up to order β belong to $L_2(\partial S, X)$, ($\|\cdot\|_{\beta,0}$ is the notation for the norm in $W^{(\beta)}(\partial S, X)$);

5. $W_{\beta,\alpha}(\partial S)$ is the Hilbert space with norm

$$\|f\|^2_{\beta,\alpha} = \|f\|^2_{0,\alpha} + \|f\|^2_{\beta,0}$$

Note. In the case where ∂S is a smooth closed curve of length l, the norm in the space $W_{\alpha,\beta}(\partial S)$ can be defined by the following method:

$$\|f\|^2_{\beta,\alpha} = \sum_{n=-\infty}^{\infty} [\|f_n\|^2_{X^\alpha} + |n|^{2\beta}\|f_n\|^2_X] \tag{3.1}$$

$$f_n = \oint_{\partial S} f\bar{e}_n ds_0, \quad e_n = (2\pi)^{-1/2}\exp(ins_0), \quad s_0 = \frac{2\pi s}{l}$$

6. H is the space of vector functions $\mathbf{w}(w_1, w_2, w_3)$ having a finite energy integral

$$\|\mathbf{w}\|^2_H = \int_S \int_{-h}^{h} \sigma_{ij}(\mathbf{w})\,\varepsilon_{ij}(\mathbf{w})\, dx\, dx_3 \tag{3.2}$$

7. H_0 is the space obtained by closure of the set of vector functions $\mathbf{v}$, each of whose components $v_i \in C_\infty(S, X)$ in the metric (3.2);

8. $C_\infty(S, X)$ is the set of functions with values in X which are finite and infinitely differentiable in S.

The elements of the space H_0 possess the following obvious properties:

$$\mathbf{v}\,|_\Gamma = 0 \tag{3.3}$$

Let us define the space of generalized solutions of the problem (1.1) and (1.2).

Definition. We call the set of elements $\mathbf{u} \in H$ satisfying the condition

$$(\mathbf{u}, \mathbf{v})_H = 0, \quad \mathbf{v} \in H_0 \tag{3.4}$$

the space of generalized solutions H_1.

In other words, H_1 is the orthogonal complement to H_0 in the metric (3.2) or $H = H_0 \oplus H_1$.

If (3.4) is integrated by parts and the property (3.3) is taken into account, we see that $\mathbf{u}$ satisfies the relationships (1.1) and (1.2).

4. We turn to a study of the question of the completeness of the system of homogeneous solutions in the space H_1.

First of all we note that on the basis of the triangle inequality we have from the general representation of the solution (1.3)

$$\|\mathbf{u}\|_H \leqslant \|\mathbf{u}^{(1)}(\Phi_0, \Phi_1)\|_H + \|\mathbf{u}^{(2)}(c)\|_H + \|\mathbf{u}^{(3)}(g)\|_H = \tag{4.1}$$
$$\|\mathbf{u}^{(1)}(\Phi_0, \Phi_1)\|_H + \|c\|_{Z_1} + \|g\|_{Z_2}$$

Here Z_1 and Z_2 are spaces induced by the energy metric for the functions c and g and the norms

$$\|c\|_{Z_1}^2 = \frac{1}{2}\int_S\int_{-h}^{h} \{p(x_3)[|\Delta c''|^2 + |\Delta^2 c|^2 - 2r(x_3)\Delta c''\Delta^2 c + \tag{4.2}$$
$$2q(x_3)[|\partial_1\Delta c'|^2 + |\partial_2\Delta c'|^2] + 2\mu(x_3)[|\partial_1\partial_2 b|^2 - \partial_1^2 b\partial_2^2 b]\}\, dx\, dx_3$$
$$b = p(x_3)c'' - q(x_3)\Delta c$$

$$\|g_*\|_{Z_2}^2 = \frac{1}{2}\int_S\int_{-h}^{h} \mu(x_3)[|\partial_1^2 g_*|^2 + |\partial_2^2 g_*|^2 + |\partial_1 g_*'|^2 + |\partial_2 g_*'|^2]\, dx\, dx_3 \tag{4.3}$$

$$g_* = g - \frac{1}{\mu^{(0)}}\int_{-h}^{h} \mu(x_3) g dx_3$$

The problem will evidently be solved if it is proved that for any $\varepsilon > 0$ and for any solutions of Eq.(1.7) $c \in Z_1$ and for (1.8) $g \in Z_2$, there exist such K and N, that

$$\left\|c - \sum_{k=1}^{K} c_k\right\|_{Z_1} < \varepsilon, \quad \left\|g_* - \sum_{n=1}^{N} g_n\right\|_{Z_2} < \varepsilon \tag{4.4}$$

The proof of the first of the inequalities (4.4) is based on the use of some a priori estimate which we present without proof because of insufficient space.

Let us consider the boundary value problem

$$L(\Delta)\, c(x) = 0, \quad c|_{\partial S} = f_1, \quad \Delta c|_{\partial S} = f_2 \tag{4.5}$$

Lemma 1. To solve the boundary value problem (4.5) there holds the following a priori estimate

$$\|c\|_{Z_1}^2 \leqslant A[\|f_1\|_{^7/_8, ^7/_8}^2 + \|f_2\|_{^3/_8, ^3/_8}^2] \tag{4.6}$$

We introduce the space of the pairs $Y^{\alpha\gamma} = X^\alpha \oplus X^\gamma$, whose elements will be denoted by $\Theta = \{\theta_1, \theta_2\}$.

We also introduce the space

$$\Pi_{\beta\alpha\delta\gamma} = W_{\beta,\alpha}(\partial S) \oplus W_{\delta,\gamma}(\partial S)$$

The elements of this space are pairs of functions $E = \{f_1, f_2\}$ defined on Γ. We define the scalar product in this space as follows:

$$(E^{(1)}, E^{(2)})_{\beta\alpha\delta\gamma} = (f_1^{(1)}, f_1^{(2)})_{\beta,\alpha} + (f_2^{(1)}, f_2^{(2)})_{\delta,\gamma} \tag{4.7}$$

where the scalar products on the right are defined by the relationship (3.1).

Let Θ_k be some complete system in the space $Y^{\alpha\beta}$, and e_n an orthonormalized basis in $L_2(\partial S)$. The following assertion is evident.

Lemma 2. The system $E_{nk} = e_n \otimes \Theta_k$ is complete in the space $\Pi_{\beta\alpha\delta\gamma}$.

Now, let us admit that the solution of (1.7) is $c \in Z_1$ and $\Gamma(c) = \{c|_{\partial S} = f_1, \Delta c|_{\partial S} = f_2\}$ is its trace on the side surface Γ. On the basis of the inequality (4.6) we have

$$\|c\|_{Z_1}^2 \leqslant A\|\Gamma(c)\|_{\alpha\beta\gamma\delta}^2 \quad (\alpha = \beta = {}^3/_8, \gamma = \delta = {}^7/_8) \tag{4.8}$$

We set $\Theta_k^0 = \{V^{7/8}\varphi_k, \beta_k V^{3/8}\varphi_k\}$, where φ_k are the eigenvectors of a quadratic bundle (2.2) (here we assume for simplicity that there are no associated vectors). As follows from [1], the system $\{\Theta_k^0\}$ is complete in the space Y^{00}, therefore, the system $\Theta_k = \{\varphi_k, \beta_k \varphi_k\}$ is complete in the space $Y^{7/8,7/8}$. There results from Lemma 2 that for any $\varepsilon > 0$ there exist N, K and constants C_{nk} such that

$$\| E - E_{Nk} \|_{\beta\alpha\delta\gamma} \leqslant \frac{\varepsilon}{A}, \quad E_{Nk} = \sum_{n=-N}^{N} \sum_{k=1}^{K} C_{nk} (e_n \otimes \Theta_k) \qquad (4.9)$$

Let us introduce the notation

$$d_k(s) = \sum_{n=-N}^{N} C_{nk} e_n(s)$$

We examine the following system of boundary value problems in the domain S:

$$\Delta m_k - \beta_k m_k = 0, \quad m_k |_{\partial S} = d_k(s) \qquad (k = 1, 2, \ldots, K) \qquad (4.10)$$

It has been shown in [3] that there are no negative reals among β_k whereupon all the boundary value problems (4.10) are solvable uniquely.

Let us consider the expression

$$c_0 = c - c_1, \quad c_1 = \sum_{k=1}^{K} m_k(x)\varphi_k = \sum_{k=1}^{K} c_k$$

Evidently c_0 is the solution of (1.7) and

$$\Gamma(c_0) = \Gamma(c) - \Gamma(c_1) = E - E_{NK}$$

The first of the inequalities (4.4) now results from the inequalities (4.8) and (4.9).

Therefore the following theorem is proved.

Theorem 1. Every solution of (1.7) can be approximated by elementary solutions of the first kind in the metric (4.2).

Note. If the system $\{\Theta_k\}$ is the basis in the space $Y^{\alpha\gamma}$, the inequality (4.4) can be understood in the sense of convergence, i.e. the elementary solutions of the first kind possess basis properties.

Because the system of eigenfunctions $\{l_t\}$ comprises an orthonormalized basis of the space X_μ, it is considerably simpler to prove the following theorem.

Theorem 2. Every solution of (1.8) belonging to the space Z_2 can be represented as a series in elementary solutions of the second kind, which converges in the metric of this space.

The following fundamental theorem results from Theorems 1 and 2 and from the inequality (4.1).

Theorem 3. The system of homogeneous solutions is complete in the space H_1.

5. Let us consider the problem of plate deformation under the effect of forces $\mathbf{t} = \{t_1, t_2, t_3\}$ applied to the side surface Γ. The following boundary condition is now added to the conditions (1.1) and (1.2):

$$n_j \sigma_{ij}(\mathbf{u}) |_\Gamma = t_i \qquad (i, j = 1, 2, 3) \qquad (5.1)$$

where n_j are the components of the exterior normal to the surface Γ.

Definition. We call the vector-function $\mathbf{u} \in H_1$ and satisfying the following integral identity

$$(\mathbf{u}, \boldsymbol{\eta})_H = Q(\boldsymbol{\eta}) \equiv \int_\Gamma t_i \eta_i \, d\Gamma, \quad \forall \boldsymbol{\eta} \in H_1 \qquad (5.2)$$

a solution of the boundary value problem (1. 1), (1. 2) and (1. 6).

Note. 1°. If the vector of solid displacement

$$\eta = \xi + \omega \times \mathbf{r} \tag{5.3}$$

is taken as η, where ξ, ω are arbitrary constant vectors, $\mathbf{r}$ is a radius-vector of a point, then since the left side of (5. 2) vanishes identically, we obtain the known necessary conditions for solvability, denoting the requirement for compliance with the equilibrium conditions

$$\int_\Gamma t_i d\Gamma = 0, \qquad \int_\Gamma (t_i x_j - t_j x_i)\, d\Gamma = 0$$

2°. The metric (3. 2) in the space H_1 only defines the half-norm since every vector function of the form (5. 3) makes the energy integral (3. 2) vanish. Hence, the question of the existence of a generalized solution reduces to studying the continuity conditions for the functional $Q(\eta)$ and the factor space $G = H_1/D$, where D is the kernel of Γ_1, i.e. a set of vector functions of the form (5. 3).

The initial problem of elasticity theory is reduced to an infinite system in [3, 6, 7], which is obtained if elementary solutions are substituted successively into the identity (4. 2) in place of η and $\delta\Phi_i, \delta\partial_n\Phi_i$, δm_k, δn_t are considered independent variations. It can be concluded on the basis of Theorem 3 that the system obtained in such manner is equivalent to the initial boundary value problem.

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ON THE NONSTATIONARY MOTION OF AN ELASTIC SPACE WITH A CRACK

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A nonstationary three-dimensional problem on the motion of an isotropic elastic medium in the presence of a crack along the half-plane is considered. Instantaneous concentrated normal and tangential pulses act at the initial instant on both edges of the half-plane. The solution for the time-periodic problem is determined by the Wiener-Hopf method, which was applied in the theory of wing vibrations [1] although the process of solving (and formulating) the problem in [1] differs from the course of the solution in this paper. Furthermore, an inverse time transformation is carried out which permits finding the solution of the nonstationary problem in the whole space at once, in the Smirnov-Sobolev form.

The problems of unsteady motion of an elastic continuous medium have been considered in [2 — 5]. The solution of a number of mixed dynamic problems for a liquid or elastic medium is given in [1, 3, 6, 7].

1. The equations of motion in displacements for an isotropic medium in the absence of body forces in the three-dimensional case are

$$\partial^2\mathbf{v}/\partial t^2 = (a^2 - b^2)\nabla\theta + b^2\nabla^2\mathbf{v}, \quad \theta = \nabla\mathbf{v}, \quad \mathbf{v} = \{v_1, v_2, v_3\} \qquad (1.1)$$

Let us initially consider the following time-periodic singular boundary value problem for a semi-infinite slit $(z = 0, -\infty < (\infty > x$

$$\sigma_{zz} = \rho\left[(a^2 - 2b^2)\theta + 2b^2\frac{\partial v_3}{\partial z}\right] = P\delta(x + x_0)\delta(y + y_0)\exp(-i\omega t) \qquad (1.2)$$

$$y < 0, \quad v_3 = 0, \quad y > 0$$

$$\sigma_{xz} = \rho b^2\left(\frac{\partial v_1}{\partial z} + \frac{\partial v_3}{\partial x}\right) = Q\delta(x + x_0)\delta(y + y_0)\exp(-i\omega t)$$

$$\infty < y < \infty$$

$$\sigma_{yz} = \rho b^2\left(\frac{\partial v_2}{\partial z} + \frac{\partial v_3}{\partial y}\right) = V\delta(x + x_0)\delta(y + y_0)\exp(-i\omega t)$$

$$\infty < y < \infty$$

$$v_{1,2,3} = O(R_1^{1/2}), \quad R_1 = \sqrt{y^2 + z^2} \to 0 \quad \text{(condition on the edge)}$$

Here x_0, y_0 are positive constants, $\delta(x)$ is a delta function, ρ is the density of the